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A Penalty Method for Solving Linear Complementarity Problem under Intuitionistic Fuzzy Environment

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Abstract: In this paper Penalty Method is proposed to solve the Fuzzy Linear Complementarity Problems (FLCP). This paper presents a new method to solve fuzzy quadratic programming problems; The Penalty Method can be classified into two methods. They are, High Penalty Method and Low Penalty Method. In this article a Fuzzy Linear Complementarity Problem is solved by both the methods and verifies which is easier and best. Then we solve the Fuzzy Linear Complementarity Problem with Symmetric Intuitionistic Trapezoidal Fuzzy numbers. The effectiveness of the proposed methods is illustrated by means of a Numerical example.

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I. INTRODUCTION

Most of the practical problems cannot be represented by linear programming model. So we made to develop more general mathematical programming methods and many significant advances have been made in the area of nonlinear programming[3]. The first major development was the fundamental paper by Kuhn-Tucker in 1951 which laid the foundations for a good deal of later work in nonlinear programming. The linear complementarity problem (LCP) is a well known problem in mathematical programming and it has been studied by many researchers. In 1968, Lemke [6] proposed a complementarity pivoting algorithm for solving linear complementarity problems. Since, the KKT conditions for quadratic programming problems can be written as a LCP,

Given the n×n matrix M and the n-dimensional vector q, the Linear Complementarity Problem (LCP) consists in finding non – negative vectors w and z which satisfy

$$w - Mz = q \tag{1.1}$$

$$w_i, z_i \geq 0, \text{ for } i = 1,2,\ldots n \tag{1.2}$$

$$\text{and } w_i z_i = 0, \quad \text{for } i = 1,2,\ldots n \tag{1.3}$$

Given the non negativity of the vectors w and z, the equation (1.3) requires that $w_i z_i = 0$ for $i = 1,2,\ldots,n$. Two such vectors are said to be complementarity. A solution (w,z) to the LCP is called a complementarity feasible solution, if it is a basic feasible solution to (1.1) and (1.2) with one of the pairs (w_i,z_i) is basic

The Linear Complementarity Problem arises in economics, engineering, game theory, and mathematical programming, and is well discussed in [4,5]. A solution to this problem is immediately available upon inspection when q is non negative, since we can set
w = q and z = 0. When referring to our methods we implicitly assume that q has at least one negative coordinate.

Lemke's algorithm [6] can be used to solve quadratic programs.
Since then the study of complementarity problems has been expanded enormously. Also, iterative methods developed for solving LCPs hold great promise for handling very large scale linear programs which cannot be tackled with the well known simplex method because of their large size and the consequent numerical difficulties. In a recent review, Pankaj Gupta et al [9] gave a fuzzy approximation to an infeasible generalized linear complementarity problem.

This paper provides a new technique for solving Intuitionistic fuzzy quadratic programming problem by converting it into a Intuitionistic fuzzy linear complementarity problem. Also this paper provides a new method of carrying out the fuzzy complementary Penalty method.

This paper is organized as follows: Section 2 provides some basic idea about the Symmetric Trapezoidal intuitionistic fuzzy number with arithmetic operations, Intuitionistic Fuzzy Linear Complementarity Problem is described in Section 3. Section 4 deals with a Penalty method, both Maximum Penalty Method and Minimum Penalty Method. In section 5, the effectiveness of the proposed method is illustrated by different example. Finally in section 6, we conclude the paper.

II. PRELIMINARIES

2.1 Fuzzy set

A Fuzzy set Ã is defined by Ã = {x,μ_A(x)}; x€A, μ_A(x)€[0,1]. In the pair (x, μ_A(x)), the first element x belong to the classical set A, the second element μ_A(x), belong to the interval [0,1] called membership function.

2.2 Symmetric Trapezoidal intuitionistic fuzzy number (STIFN)

A Symmetric Trapezoidal intuitionistic fuzzy number (STIFN) is an intuitionistic fuzzy set in R with the following function $\mu_{\tilde{A}^I(X)}$ and non-membership function $\upsilon_{\tilde{A}^I(X)}$.

$$\mu_{\tilde{A}^I} = \begin{cases} 0 & , x < a_1 - h \\ \dfrac{x-(a_1-h)}{h} & , a_1 - h \le x \le a_1 \\ 1 & , a_1 \le x \le a_2 \\ \dfrac{(a_2+h)-x}{h} & , a_2 \le x \le a_2 + h \\ 0 & , a_2 + h \le x \end{cases} \qquad \mu_{\tilde{A}^I} = \begin{cases} 1 & , x < a_1 - h' \\ \dfrac{x-(a_1-h')}{h'} & , a_1 - h' \le x \le a_1 \\ 0 & , a_1 \le x \le a_2 \\ \dfrac{(a_2+h')-x}{h'} & , a_2 \le x \le a_2 + h' \\ 1 & , a_2 + h' \le x \end{cases}$$

Where $a_1 \le a_2$ and $h, h' \ge 0$. This STIFN is denoted by $\tilde{A}^I = \{(a_1, a_2, h, h); (a_1, a_2, h', h')\}$. for our Convenience. STIFN is denoted by $\tilde{A}^I = (a_1, a_{2,} h, h')$ throughout this paper.

Definition: 2.3. Modified arithmetic Operations on Symmetric trapezoidal intuitionistic fuzzy numbers STIFNS)

Let $\tilde{A}^I = (a_1, a_2, h, h')\, and\, \tilde{B}^I = (b_1, b_2, k, k')$ be two Symmetric trapezoidal intuitionistic fuzzy numbers.

Then **(i) Addition:** $\tilde{A}^I + \tilde{B}^I = (a_1 + b_1, a_2 + b_2, h + k, h' + k')$

(ii) Subtraction: $\tilde{A}^I - \tilde{B}^I = (a_1 - b_2, a_2 - b_1, h + k, h' + k')$

(iii) Multiplication: $\tilde{A}^I \times \tilde{B}^I = (\frac{a_1 + a_2}{2} \times \frac{b_1 + b_2}{2} - W), (\frac{a_1 + a_2}{2} \times \frac{b_1 + b_2}{2} - W.|w - w'|.|w - w_1'|)$

Where $w = \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right) - \min(m).\max(m) - \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right)$

$$w' = \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right) - \min(n).\max(n) - \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right)$$

$$w_1' = \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right) - \min(n').\max(n') - \left(\frac{a_1 + a_2}{2}\right)\left(\frac{b_1 + b_2}{2}\right), \quad m = (a_1 b_1, a_1 b_2, a_2 b_{1,} a_2, b_2)$$

$$n = (a_1 - h)(b_1 - k).(a_1 - h)(b_1 + k).(a_2 + h)(b_1 - k).(a_2 + h)(b_2 + k),$$

$$n' = (a_1 - h')(b_1 - k').(a_1 - h')(b_1 + k').(a_2 + h')(b_1 - k').(a_2 + h')(b_2 + k')$$

iv) Division: $\tilde{A}^I \times \tilde{B}^I = (\frac{a_1 + a_2}{b_1 + b_2} - w, \frac{a_1 + a_2}{b_1 + b_2} + .|w - w'|.|w - w_1'|)$

Where $w = \min\left\{\left[\frac{a_1 + a_2}{b_1 + b_2}\right] - \min(m), \max(m) - \left[\frac{a_1 + a_2}{b_1 + b_2}\right]\right\},$

$$w' = \min\left\{\left[\frac{a_1+a_2}{b_1+b_2}\right] - \min(n), \max(n) - \left[\frac{a_1+a_2}{b_1+b_2}\right]\right\},$$

$$w'_1 = \min\left\{\left[\frac{a_1+a_2}{b_1+b_2}\right] - \min(n'), \max(n') - \left[\frac{a_1+a_2}{b_1+b_2}\right]\right\}, \quad m = \left(\frac{a_1}{b_1}, \frac{a_1}{b_2}, \frac{a_2}{b_1}, \frac{a_2}{b_2}\right),$$

$$n = \left(\left[\frac{a_1-h}{b_1-k}\right], \left[\frac{a_1-h}{b_2+k}\right], \left[\frac{a_2+h}{b_1-k}\right], \left[\frac{a_2+h}{b_2+k}\right]\right), \; n' = \left(\left[\frac{a_1-h'}{b_1-k'}\right], \left[\frac{a_1-h'}{b_2+k'}\right], \left[\frac{a_2+h'}{b_1-k'}\right], \left[\frac{a_2+h'}{b_2+k'}\right]\right),$$

Definition: 2.4. Let F(S) be the set of all Symmetric trapezoidal intuitionistic fuzzy numbers. For $\tilde{A}^{I} = \{(a_1, a_2, h, h); (a_1, a_2, h', h')\} \in \Re(s)$ we define a ranking function $F: F(s) \to \Re$ by $F(\tilde{A}^{I}) = [\frac{(a_1+a_2}{2} + (h-h')]$.

III. INTUITIONISTIC FUZZY LINEAR COMPLEMENTARITY PROBLEM (IFLCP)

3.1. Fuzzy Linear Complementarity Problem (FLCP)

Assume that all parameters in (1) - (3) are fuzzy and are described by fuzzy numbers. Then, the following fuzzy linear complementarity problem[5] can be obtained by replacing crisp parameters with fuzzy numbers.

$$\tilde{W} - \tilde{M}\tilde{Z} = \tilde{q} \tag{3.1}$$

$$\tilde{W}_j \geq 0, Z_j \geq 0, j = 1,2,3,.........n \tag{3.2}$$

$$\tilde{W}_j \tilde{Z}_j = 0, j = 1,2,3,...........n \tag{3.3}$$

The pair $(\tilde{W}_j, \tilde{Z}_j)$ is said to be a pair of fuzzy complementary variables.

3.2 Intuitionistic Fuzzy Quadratic Programming Problem (QPP) as a Intuitionistic fuzzy Linear Complimentarity Problem (LCP)

Consider the following Quadratic Programming Problem

Minimize $\tilde{f}^{I}(\tilde{x})^{I} = \tilde{c}^{tI}\tilde{x}^{I} + \frac{1}{2}\tilde{x}^{tI}\tilde{H}^{I}\tilde{x}^{I}$

Subject to $\tilde{A}^{I}\tilde{x}^{I} \leq \tilde{b}^{I}$ and $\tilde{x}^{I} \geq 0$

Where $\tilde{c}$ an n-vector of fuzzy numbers is, $\tilde{b}$ is an m-vector, Ã is an mxn fuzzy matrix and $\tilde{H}$ is an nxn fuzzy symmetric matrix. Let $\tilde{y}$ denotes the vector of slack variables and $\tilde{u}, \tilde{v}$ be the Lagrangian[9] multiplier vectors of the constraints Ã $\tilde{x} \leq \tilde{b}$ and $\tilde{x} \geq 0$ respectively, then the Kuhn-Tucker conditions can be written as

Ã$^{I}\tilde{x}^{I} + \tilde{y}^{I} = \tilde{b}^{I}$ - $\tilde{H}^{I}\tilde{x}^{I} - \tilde{A}^{tI}\tilde{u}^{I} + \tilde{v}^{I} = \tilde{c}^{I}$ $\tilde{x}^{tI}\tilde{v}^{I} = \tilde{0}^{I}, \tilde{u}^{tI}\tilde{y}^{I} = \tilde{0}^{I}$ And $\tilde{x}^{I}, \tilde{y}^{I}$ $\tilde{u}^{I}, \tilde{v}^{I} \geq \tilde{0}^{I}$ Now Letting $\tilde{M}^{I} = \begin{bmatrix} \tilde{0}^{I} & -\tilde{A}^{I} \\ \tilde{A}^{tI} & \tilde{H}^{I} \end{bmatrix}, \tilde{q}^{I} = \begin{bmatrix} \tilde{b}^{I} \\ \tilde{c}^{I} \end{bmatrix}, \tilde{w}^{I} = \begin{bmatrix} \tilde{y}^{I} \\ \tilde{v}^{I} \end{bmatrix}$ and $\tilde{z}^{I} = \begin{bmatrix} \tilde{u}^{I} \\ \tilde{v}^{I} \end{bmatrix}$ the Kuhn-Tucker

conditions can be expressed as the LCP. $\tilde{W}^{I} - \tilde{M}^{I}\tilde{Z}^{I} = \tilde{q}^{I}, \tilde{W}^{tI}\tilde{Z}^{I} = \tilde{0}^{I}, (\tilde{W}^{I}, \tilde{Z}^{I}) \geq \tilde{0}^{I}$

IV. PENALTY METHOD

Penalty methods are a certain class of algorithms for solving constrained optimization problems. A **penalty method** replaces a constrained optimization problem by a series of unconstrained problems whose solutions ideally converge to the solution of the original constrained problem.

W.I.Sangwill [2,10] suggested a Penalty method for solving Non Linear Programming Problems. Based on this idea, the methods for solving Intuitionistic[1] fuzzy linear complementarity problem are developed here. This Penalty method can be classified into two Types; these are High Penalty Method and Low Penalty Method.

4.1 High penalty Method:

Step 0: Initialization: Input $q^0, M^0 = q, M$ with M € P,Set t = 0.

Step 1: Test for Termination: If $q^t \geq 0$, then stop. $z^t = 0$ Solves q^t, M^t. That is $w, z = q, 0$.

Step 2: Choose Pivot Row: Choose the Penalty p, so that $p = Max\, i; q_i^{\,t} < 0.$

Step 3: Pivoting: Pivot on $m^t{}_{pp}$.

4.2 Low penalty Method:

Step 0: Initialization: Input $q^0, M^0 = q, M$ with M € P,Set t = 0.

Step 1: Test for Termination: If $q^t \geq 0$, then stop. $z^t = 0$ Solves q^t, M^t. That is $w, z = q, 0$.

Step 2: Choose Pivot Row: Choose the Penalty p, so that $p = Min\, i; q_i^{\,t} < 0.$

Step 3: Pivoting: Pivot on $m^t{}_{pp}$.

The pivot operations in both the method is similar to the simplex method pivot operations. If $m^t{}_{pp}$ =0, then the above two methods are unable to solve this LCP.

Theorem: Let q, M be an FLCP of order n with M € P. Then for any $q \in R^n$, the above two methods will solve the FLCP q, M in a finite number of steps. Furthermore, no Complementarity basis will ever be used more than once.

Proof:

The proof is by induction on n.For n = 1, the theorem is obvious. In this case at most one pivot step is required. Inductively, assume that n > 1 and that the theorem holds for all Intuitionistic fuzzy linear Complementarity Problems (Of the P – Matrix type [7,8]) of order less than or equal to n – 1.

The FLCP q, M under consideration has a unique solution z. There are two main cases, according to whether $z_n = 0$ or not.

Case 1: $z_n = 0$. Suppose the above method is applied to the leading principal sub problem of order n – 1. By the inductive hypothesis, the method obtains the unique solution $z \in R^{n-1}$ of this problem in a finite number of steps, during the course of which no complimentarity basis is repeated. Therefore in this case, the theorem holds.

Case 2: $z_n > 0$, Applying the above method to q, M , we arrive after a finite number of steps at a complementarity basis such that $(C^{-1}q)_i \geq 0$ for i = 1, 2,…, n-1 and $(C^{-1}q)_i < 0$. In other words, the leading principal sub problem of order n – 1 has been solved without repetition of a complementarity basis.

The method then calls for a pivot on the last diagonal element of the current principal pivotal transform of The new principal pivotal transform of q, M is a linear complementarity problem of the P – matrix type having the property covered in case 1. Accordingly, when the algorithm continues, it solves this new FLCP in a finite number of steps without repeating a complementarity basis. Hence it solves the original problem q, M in a finite number of steps.

V. NUMERICAL EXAMPLES

5.1 Example: 1

Consider the following FLC problem

$$\tilde{M}^{I} = \begin{bmatrix} -\tilde{1}^{I} & \tilde{1}^{I} & \tilde{1}^{I} & \tilde{1}^{I} \\ \tilde{1}^{I} & -\tilde{1}^{I} & \tilde{1}^{I} & \tilde{1}^{I} \\ -\tilde{1}^{I} & -\tilde{1}^{I} & -\tilde{2}^{I} & \tilde{0}^{I} \\ -\tilde{1}^{I} & -\tilde{1}^{I} & \tilde{0}^{I} & -\tilde{2}^{I} \end{bmatrix} \text{ and } \tilde{q}^{I} = \begin{bmatrix} \tilde{3}^{I} \\ \tilde{5}^{I} \\ -\tilde{9}^{I} \\ -\tilde{5}^{I} \end{bmatrix}$$

Now, the fuzzy linear complementary problem is solved by the proposed algorithm and the results are tabulated below.

Type:1 High Penalty method

TABLE: 1

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(2,4,2,2)
W_2	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	-(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(3,7,1,1)
W_3	(0,0,0,0)	(0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,1,0,0)	-(1,1,0,0)	-(1,3,2,2)	(0,0,0,0)	-(2,10,3,3)
W_4	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	-(1,1,0,0)	-(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)*	-(3,7,1,1)
C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1/2,1/2,0,0)	-(3/2,3/2,1,1)	(1/2,1/2,0,0)	(1,1,0,0)	(0,0,0,0)	(1/2,1/2,0,0)
W_2	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	-(3/2,3/2,1,1)	(1,1,0,0)	(0,0,0,0)	(5/2,5/2,2,2)
W_3	(0,0,0,0)	(0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,1,0,0)	-(1,1,0,0)	-(1,3,2,2)*	(0,0,0,0)	-(2,10,3,3)
Z_4	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	-(1/2,1/2,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	(0,0,0,0)	(1,1,0,0)	(5/2,5/2,2,2)
C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,0,0)	(0,0,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	-(1,3,2,2)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	-(4,4,1,1)
W_2	(0,0,0,0)	(1,1,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	(0,0,0,0)	-(1,3,2,2)*	(0,0,0,0)	(0,0,0,0)	-(1,3,2,2)
Z_3	(0,0,0,0)	(0,0,0)	-(1/2,1/2,0,0)	(0,0,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	(1,1,0,0)	(0,0,0,0)	(9/2,9/2,1/2,1/2)
Z_4	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	-(1/2,1/2,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	(0,0,0,0)	(1,1,0,0)	(5/2,5/2,2,2)

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,0,0)	(0,0,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	-(1,3,2,2)*	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	-(4,4,1,1)
Z_2	(0,0,0,0)	-(1/2,1/2,0,0)	-(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
Z_3	(0,0,0,0)	(1/4,1/4,0,0)	-(3/8,3/8,0,0)	(1/8,1/8,0,0)	(1/2,1/2,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(4,4,0,0)
Z_4	(0,0,0,0)	(1/4,1/4,0,0)	(1/8,1/8,0,0)	-(3/8,3/8,0,0)	(1/2,1/2,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(1,3,2,2)
C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
Z_1	-(1/2,1/2,0,0)	(0,0,0,0)	-(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,3,2,2)
Z_2	(0,0,0,0)	-(1/2,1/2,0,0)	-(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
Z_3	(1/4,1/4,0,0)	(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(1/4,1/4,0,0)	(0,0,0,0)	(0,0,0)	(1,1,0,0)	(0,0,0,0)	(2,4,1,1)
Z_4	(1/4,1/4,0,0)	(1/4,1/4,0,0)	(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(1,1,0,0)

Thus $\{W_1, W_2, W_3, W_4 ; Z_1 , Z_2, Z_3, Z_4\}$ = {(0,0,0,0), (0,0,0,0), (0,0,0,0), (0,0,0,0); (1,3,2,2), (1,1,0,0), (2,4,1,1), (1,1,0,0)} is a Complimentarity feasible Solution of this IFLCP

Type2: Low Penalty method

TABLE : 2

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(2,4,2,2)
W_2	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	-(1,1,0,0)	(1,1,0,0)	(1,1,0,0)	(3,7,1,1)
W_3	(0,0,0,0)	(0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,1,0,0)	-(1,1,0,0)	-(1,3,2,2)*	(0,0,0,0)	-(2,10,3,3)
W_4	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	-(1,1,0,0)	-(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)	-(3,7,1,1)
C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
W_1	(1,1,0,0)	(0,0,0,0)	(1/2,12,0,0)	(0,0,0,0)	-(3/2,3/2,0,0)*	(1/2,12,0,0)	(0,0,0,0)	(1,1,0,0)	-(3/2,3/2,0,0)
W_2	(0,0,0,0)	(1,1,0,0)	(1/2,12,0,0)	(0,0,0,0)	(1/2,12,0,0)	-(3/2,3/2,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)
Z_3	(0,0,0,0)	(0,0,0)	-(1/2,12,0,0)	(0,0,0,0)	(1/2,12,0,0)	(1/2,12,0,0)	(1,1,0,0)	(0,0,0,0)	(9/2,9/2,0,0)
W_4	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	-(1,1,0,0)	-(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)	-(3,7,1,1)
C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
Z_1	-(2/3,2/3,0,0)	(0,0,0,0)	-(1/3,1/3,0,0)	(0,0,0,0)	(1,1,0,0)	-(1/3,1/3,0,0)	(0,0,0,0)	-(2/3,2/3,0,0)	(1,1,0,0)
W_2	(1/3,1/3,0,0)	(1,1,0,0)	(2/3,2/3,0,0)	(0,0,0,0)	(0,0,0,0)	-(4/3,4/3,0,0)	(0,0,0,0)	(4/3,4/3,0,0)	(0,0,0,0)
Z_3	(1/3,1/3,0,0)	(0,0,0,0)	-(1/3,1/3,0,0)	(0,0,0,0)	(0,0,0,0)	(2/3,2/3,0,0)	(1,1,0,0)	(1/3,1/3,0,0)	(2,6,1,1)
W_4	-(2/3,2/3,0,0)	(0,0,0,0)	-(1/3,1/3,0,0)	(1,1,0,0)	(0,0,0,0)	-(4/3,4/3,0,0)	(0,0,0,0)	-(8/3,8/3,0,0)*	-(2,6,1,1)
C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
Z_1	-(1/2,1/2,0,0)	(0,0,0,0)	-(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,3,1,1)
W_2	(0,0,0,0)	(1,1,0,0)	(1/2,1/2,0,0)	(1/2,1/2,0,0)	(0,0,0,0)	-(1,3,1,1)*	(0,0,0,0)	(0,0,0,0)	-(1,3,1,1)
Z_3	(1/4,1/4,0,0)	(0,0,0,0)	-(3/8,3/8,0,0)	(1/8,1/8,0,0)	(0,0,0,0)	(1/2,1/2,0,0)	(1,1,0,0)	(0,0,0,0)	(7/27/2,0,0)
Z_4	(1/4,1/4,0,0)	(0,0,0,0)	(1/8,1/8,0,0)	-(3/8,3/8,0,0)	(0,0,0,0)	(1/2,1/2,0,0)	(0,0,0,0)	(1,1,0,0)	(3/2,3/2,0,0)

C_B	W_1	W_2	W_3	W_4	Z_1	Z_2	Z_3	Z_4	q
Z_1	-(1/2,1/2,0,0)	(0,0,0,0)	-(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,3,1,1)
Z_2	(0,0,0,0)	-(1/2,1/2,0,0)	-(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
Z_3	(1/4,1/4,0,0)	(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(1/4,1/4,0,0)	(0,0,0,0)	(0,0,0)	(1,1,0,0)	(0,0,0,0)	(1,5,1,1)
Z_4	(1/4,1/4,0,0)	(1/4,1/4,0,0)	(1/4,1/4,0,0)	-(1/4,1/4,0,0)	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(1,1,0,0)

Thus $\{W_1, W_2, W_3, W_4 ; Z_1, Z_2, Z_3, Z_4\}$ = {(0,0,0,0), (0,0,0,0), (0,0,0,0), (0,0,0,0);(1,3,1,1), (1,1,0,0), (1,5,1,1), (1,1,0,0)} is a Complimentarity feasible Solution of this IFLCP

5.2 Example: 2

Consider the following QP problem

$$\widetilde{M}^{I} = \begin{bmatrix} \widetilde{1}^{I} & \widetilde{0}^{I} & \widetilde{0}^{I} \\ \widetilde{2}^{I} & \widetilde{1}^{I} & \widetilde{0}^{I} \\ \widetilde{2}^{I} & \widetilde{2}^{I} & \widetilde{1}^{I} \end{bmatrix} \text{ and } \widetilde{q}^{I} = \begin{bmatrix} -\widetilde{1}^{I} \\ -\widetilde{1}^{I} \\ -\widetilde{1}^{I} \end{bmatrix}$$

Type 1: High penalty method

TABLE:3

C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)
W_2	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)	-(1,1,0,0)	(0,0,0,0)	-(1,1,0,0)
W_3	(0,0,0,0)	(0,0,0)	(1,1,0,0)	-(1,3,2,2)	-(1,3,2,2)	-(1,1,0,0)*	-(1,1,0,0)
C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)
W_2	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)	-(1,1,0,0)*	(0,0,0,0)	-(1,1,0,0)
Z_3	(0,0,0,0)	(0,0,0)	-(1,1,0,0)	(1,3,2,2)	(1,3,2,2)	(1,1,0,0)	(1,1,0,0)
C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)
Z_2	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(1,3,2,2)	(1,1,0,0)	(0,0,0,0)	(1,1,0,0)
Z_3	(0,0,0,0)	(1,3,2,2)	-(1,1,0,0)*	-(1,3,2,2)	(0,0,0)	(1,1,0,0)	-(1,1,0,0)
C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)*	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)
Z_2	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(1,3,2,2)	(1,1,0,0)	(0,0,0,0)	(1,1,0,0)
W_3	(0,0,0,0)	-(1,3,2,2)	(1,1,0,0)	(1,3,2,2)	(0,0,0)	-(1,1,0,0)	(1,1,0,0)
C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
Z_1	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
Z_2	(1,3,2,2)	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,1,0,0)
W_3	(1,3,2,2)	-(1,3,2,2)	(1,1,0,0)	(0,0,0,0)	(0,0,0)	-(1,1,0,0)*	-(1,1,0,0)

C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
Z_1	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
Z_2	(1,3,2,2)	-(1,1,0,0)*	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,1,0,0)
Z_3	-(1,3,2,2)	(1,3,2,2)	-(1,1,0,0)	(0,0,0,0)	(0,0,0)	(1,1,0,0)	(1,1,0,0)

C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
Z_1	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
W_2	-(1,3,2,2)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(1,1,0,0)
Z_3	-(1,3,2,2)	(0,0,0)	-(1,1,0,0)*	(0,0,0,0)	(1,3,2,2)	(1,1,0,0)	-(1,1,0,0)
C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
Z_1	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
W_2	-(1,3,2,2)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(1,1,0,0)
W_3	(1,3,2,2)	(0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)	-(1,1,0,0)	(1,1,0,0)

Thus $\{W_{1,}\ W_2,\ W_3\ ;\ Z_1,Z_2,Z_3\}$ = {(0,0,0,0), (1,1,0,0), (1,1,0,0); (1,1,0,0), (0,0,0,0) ,(0,0,0,0)} is a Complimentarity feasible Solution of this IFLCP

T**ype2: Low Penalty method:**

TABLE:4

C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
W_1	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)*	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)
W_2	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)	-(1,1,0,0)	(0,0,0,0)	-(1,1,0,0)
W_3	(0,0,0,0)	(0,0,0)	(1,1,0,0)	-(1,3,2,2)	-(1,3,2,2)	-(1,1,0,0)	-(1,1,0,0)
C_B	W_1	W_2	W_3	Z_1	Z_2	Z_3	q
Z_1	-(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	(1,1,0,0)
W_2	-(1,3,2,2)	(1,1,0,0)	(0,0,0,0)	(0,0,0,0)	-(1,1,0,0)	(0,0,0,0)	(1,1,0,0)
W_3	(1,3,2,2)	(0,0,0)	(1,1,0,0)	(0,0,0,0)	-(1,3,2,2)	-(1,1,0,0)	(1,1,0,0)

Thus $\{W_{1,}\ W_2,\ W_3\ ;\ Z_1,Z_2,Z_3\}$ = {(0,0,0,0), (1,1,0,0), (1,1,0,0); (1,1,0,0), (0,0,0,0) ,(0,0,0,0)} is a Complimentarity feasible Solution of this IFLCP

VI. CONCLUSION

In this paper, two new methods for solving the Intuitionistic fuzzy linear complementarity problem are suggested. Numerical examples are solved by both methods. In numerical example 1, both the High Penalty Method and Low Penalty Method yields same solution with same number of iterations. In numerical example 2, High Penalty Method Yield a solution at 7^{th} iteration but Low Penalty Method yields the same solution at first iteration. Hence we need not conclude that in Real Life Situations Sometimes Low Penalty Method is better than High Penalty Method, it occurs only on some time but not always.

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